An Integrated Optimization Method for Train Timetabling and Maintenance Scheduling Problem



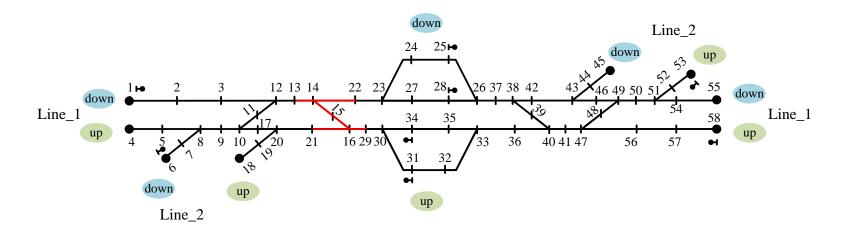


CONTENTS

- 1 Introduction
- 2 Model Formulation
- 3 Solution Approaches
- 4 Case Study
- 5 Conclusion

1 Introduction





Determining arrival and departure time of the trains on traveled links is the essential work for joint optimization on train timetabling and maintenance task scheduling problem.

Block sections traveled by the trains as well as start and end time for trains occupying block sections are treated as core decision variables.

A mixed integer linear programming model is built on this basis to solve the integrated optimization problem, and the model is solved through a commercial solver.

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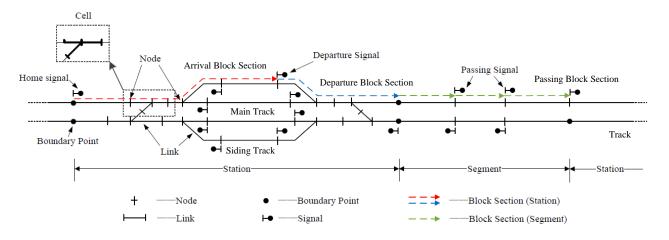
2 Model Formulation

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2.1 Definition of Sets and Parameters

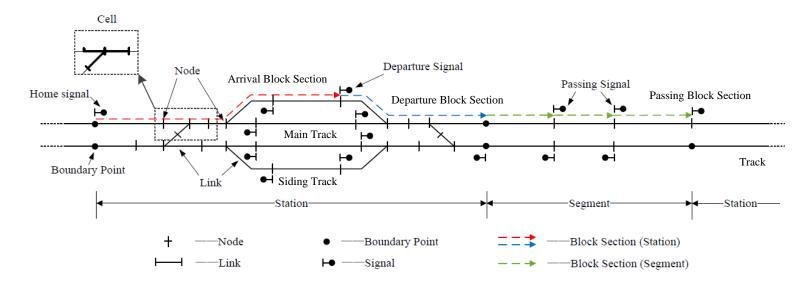
(1) Fundamental sets



- R set of trains, index by r, i.e., $r \in R$
- set of block sections, index by b, i.e., $b \in B$. Block section can be divided into three types, i.e., arrival block section, departure block section and passing block section
- set of cells, index by c, i.e., $c \in C$. Cells are the minimal units to identify the conflicts between block sections
- L set of links, index by l, i.e., $l \in L$



(1) Fundamental sets



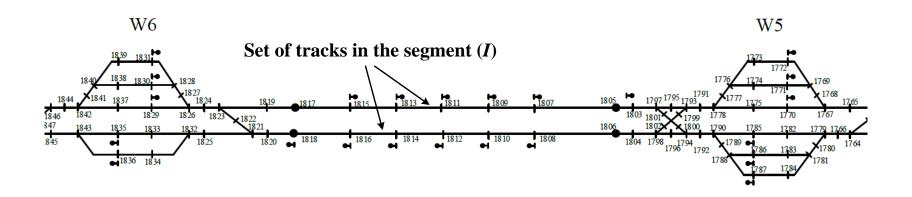
S set of stations, index by s, i.e., $s \in S$

N set of nodes, index by n, i.e., $n \in N$

MOT set of track maintenance tasks, index by m, i.e., $m \in MOT$

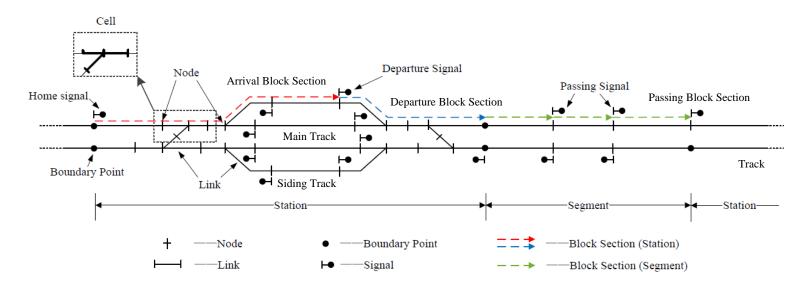


(1) Fundamental sets



set of tracks in the segments, index by i, i.e., $i \in I$. A segment consists of one double-track usually has two tracks and each of the track is made up of several passing block sections

(2) Sets and parameters related to block section



 $B^{arrival}$ set of arrival block sections, $B^{arrival} \subset B$

 B^{share} set of passing block sections shared by two or more tracks in the segments, $B^{share} \subset B$

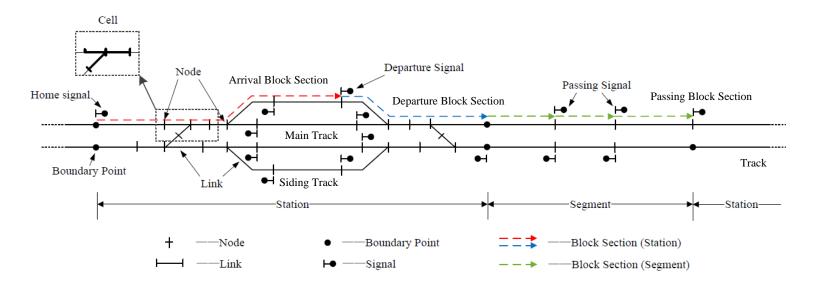
 B_r set of block sections that train r can potentially travel on, $B_r \subseteq B$

 B_n^+ , B_n^- set of block sections flow out (in) node n, B_n^+ , $B_n^- \subset B$

 B_c set of block sections containing cell $c, B_c \subset B$



(2) Sets and parameters related to block section



 B_{i_c} set of passing block sections located on the track parallel to the track i_c that cell c is located, $B_{i_c} \subseteq B$

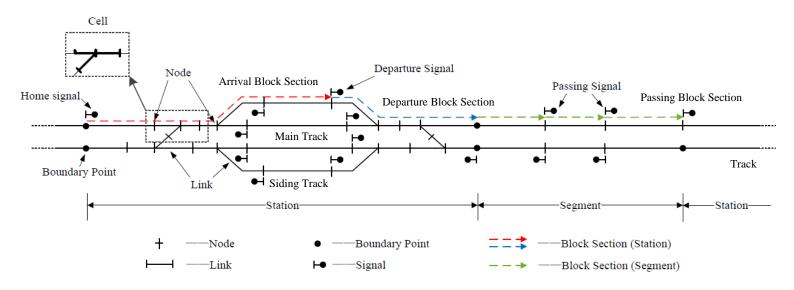
0-1 relationship parameter, equals 1 if block section b conflicts with block section b', 0 otherwise. Two block sections are conflict with each other if they have cells in common

 l_b^{last} last link of arrival block section b

 n_b^e end node of arrival block section b



(3) Sets and parameters related to node



Nsiding

set of nodes which have the same location as departure signals and are located on the siding tracks, i.e., those nodes are connections between two block sections, $N^{siding} \subset N$

Nmain

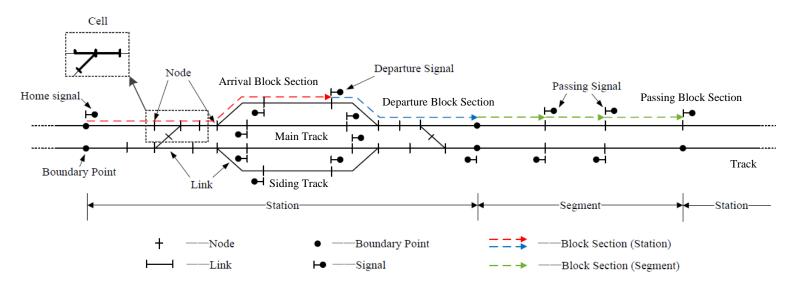
set of nodes which have the same location as departure signals and are located on the main tracks, i.e., those nodes are connections between two block sections, $N^{main} \subset N$

 $N^{boundary}$ set of nodes serve as the boundary points, $N^{boundary} \subset N$

 N_s set of nodes in station $s, N_s \subset N$



(3) Sets and parameters related to node



 N_r

set of nodes that train r can potentially travel on, the origin and destination nodes excluded, $N_r \subset N$

 $N_r^{special}$

set of special nodes that train r can potentially travel on, $N_r^{special} \subset N$

 $s_r^{special}$

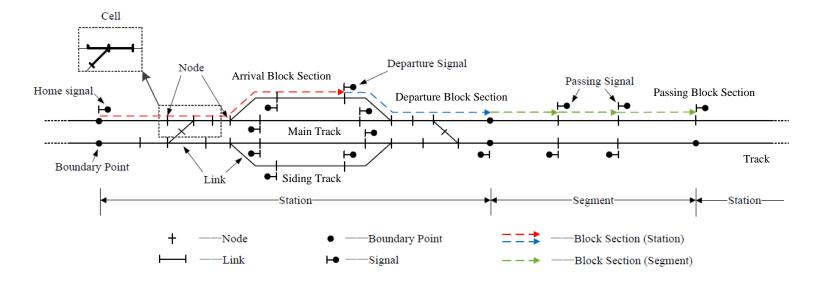
index of the special origin station for train r. In the special origin station, origin node of train r is in the home signal

 s^n

index of the station containing node n



(4) Sets and parameters related to cell

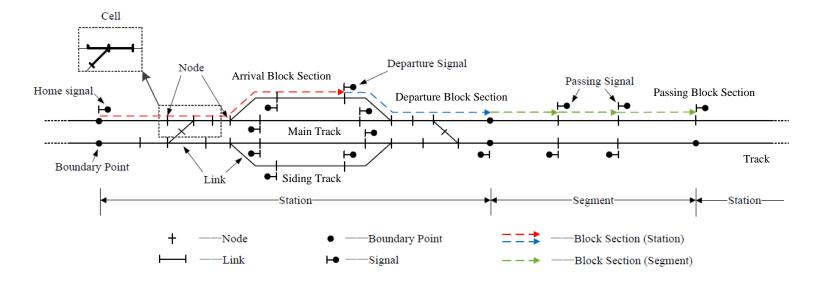


 C_b set of cells in block section $b, C_b \subset C$

 i_c index of the track for cell c



(5) Sets and parameters related to link

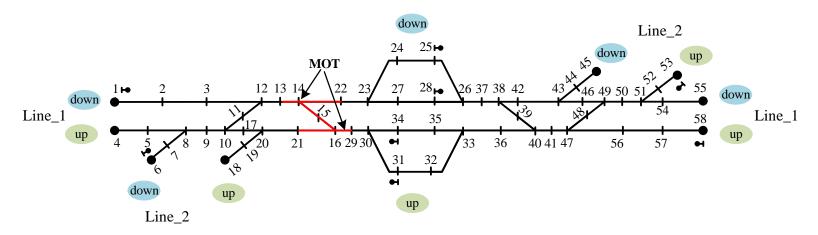


 L_b set of links for block section $b, L_b \subset L$

 L_c set of links for cell c, i.e., $L_c \subset L$



(6) Sets and parameters related to MOT



 C_m set of cells included in track maintenance task m, $C_m \subset C$ set of cells adjacent to cell c, i.e., $MOT_c^{adjacent} \subset C$. All of the cells in $MOT_c^{adjacent}$ have track maintenance tasks $[mot_m^s, mot_m^e]$ starting time window of track maintenance task m

 v_c^{limit} maximum speed of the other whole track during track

maintenance task on cell *c*

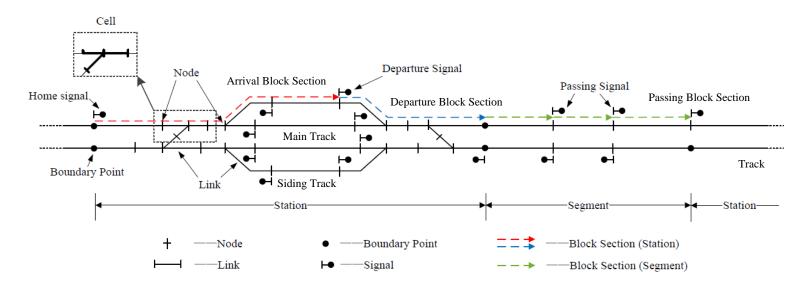
 v_c^1, v_c^2 maximum speed of the first (second) train traveling through cell c after track maintenance task on cell c is finished

minimum time duration of track maintenance task m

 d_m



(7) Sets and parameters related to train



 S_r set of stations that train r can potentially travel on, the origin and destination stations excluded, $S_r \subseteq S$

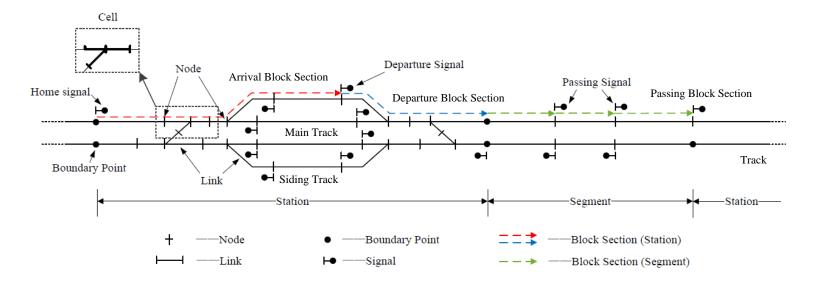
 $[t_r^s, t_r^e]$ time window that train r can leave from origin node n_r^o

 $t_{r,l}$ minimal running time of train r on link l, $t_{r,l}$ is round up to an integer when it's value is fractional

 n_r^o, n_r^d index of the origin (destination) node for train r



(7) Sets and parameters related to train



 $dwell_{r,s}$ prescribed dwell time of train r in station s s_r^o, s_r^d index of the origin (destination) station for train r l_r^v maximum speed of train r on link l

Last parameter used is the sufficiently large number M.



2.2 Definition of decision variables

(1) Decision variables related to train properties

 $y_{r,b}^{entry}$, $y_{r,b}^{exit}$ entry (exit) time for train r at block section b $x_{r,b}$ 0-1 variable, equals 1 if train r uses block section b, 0 otherwise stop_{r,s} scheduled dwell time of train r in station s

(2) Decision variables related to train sequence

0-1 variable, equals 1 if train r is scheduled earlier on block section b than train r' is scheduled on block sections b' which is in conflict with block section b, 0 otherwise

(3) Decision variables related to MOT properties

 $start_c$ start time of track maintenance task on cell c end c end time of track maintenance task on cell c



(4) Decision variables related to the relation between train and MOT

$lpha_{r.b,c}$	0-1 variable, equals 1 if entry time of train r at block section b is greater than or equal to end time of track maintenance task on cell c , 0 otherwise
$oldsymbol{eta_{r.b,c}}$	0-1 variable, equals 1 if train r can exit block section b at its maximal speed before maintenance task on cell c starts, 0 otherwise
$\mathbf{z}_{r,c}$	0-1 variable, equals 1 if train r travels through cell c after track maintenance task on cell
$\mathbf{z}_{r,c}^1$	0-1 variable, equals 1 if train r is the first train traveling through cell c after track maintenance task on cell
$\mathbf{z}_{r,c}^2$	0-1 variable, equals 1 if train r is the second train traveling through cell c after track maintenance task on cell c is finished, 0 otherwise



Release time of the arrival block section

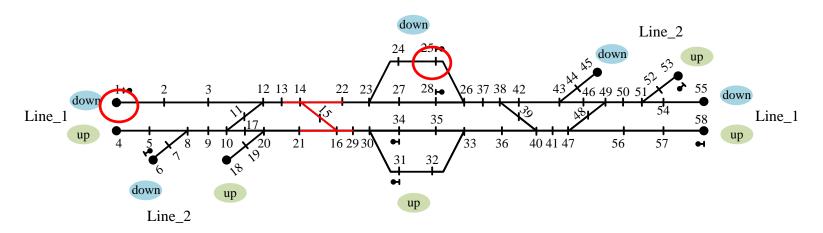


Figure 1 Illustration on releasing the arrival block section in advance

As shown in Fig.1, for the arrival block section from node 1 to node 25, when the train traverse on the node 24, the links from node 1 to node 24 could be used by the other trains except the link from 24 to 25. Hence, the release time of the arrival block section should exclude the last link movement time for the train.



Departure block section splitting

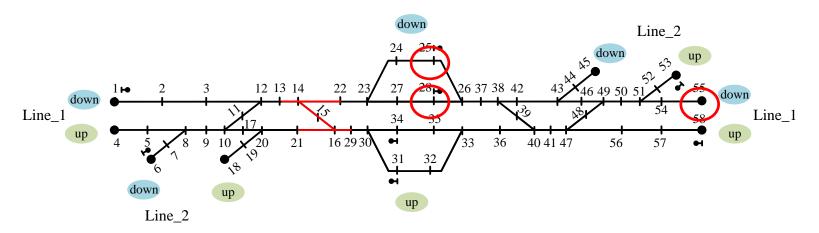


Figure 1 Illustration on releasing the arrival block section in advance

In the original data, the departure block sections are defined on the basis of cells which causes ambiguity of train routes.

In this way, we spilt a departure block section into two train routes which are made up of links. However, those two train routes share same cells as the original departure block section which means those two train routes are conflicting with each other.

20



Original location of the train

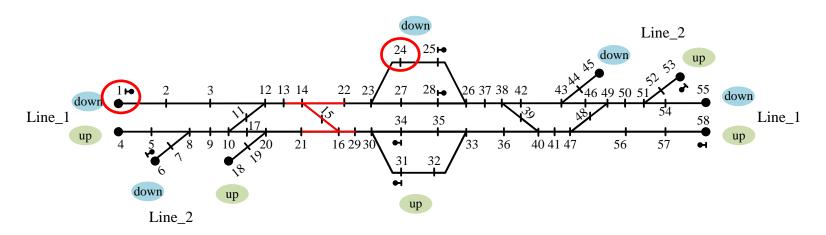


Figure 1 Illustration on releasing the arrival block section in advance

Two special situations should be noted since some trains will enter the rail network at nodes where departure signals are not located.

- (1) The train enters the system at the node behind the departure signal, such as node 24 and node 27 in Fig 1.
 - (2) The train is located on the boundary point, such as node 1 in Fig1.



Turnover movement on the links

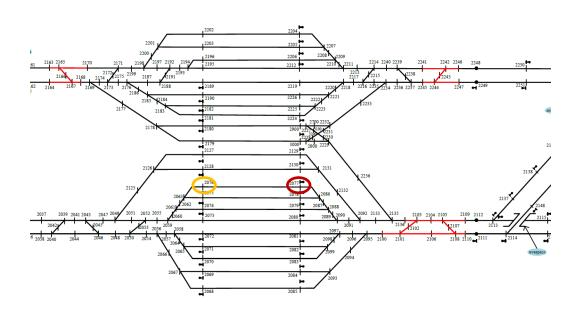


Figure 2 Station layout for train turnover movement on the links

For bi-directional links which can be passed by from both directions (up or down), trains will arrival at one node but leave from a different node. This scenario is defined as a turnover movement.

In this situation, we add a virtual block section which is similar to passing block section, so that a turnover movement can happen by passing this virtual block section.



Analysis on speed restriction on the other whole parallel track between two stations

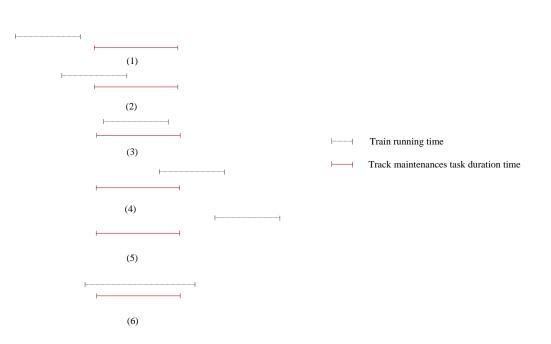


Figure 3 Analysis on speed restriction on the whole parallel track between two stations

The speed restriction constraint should be applied to every passing block section in the parallel track.

It is too complex to constrain the train's speed on each passing block section linearly and therefore, we simplify the speed constrains by assuming that train's speed on the whole passing block section will be constrained for all six situations above except situations in Fig3 (1) and Fig3 (5).

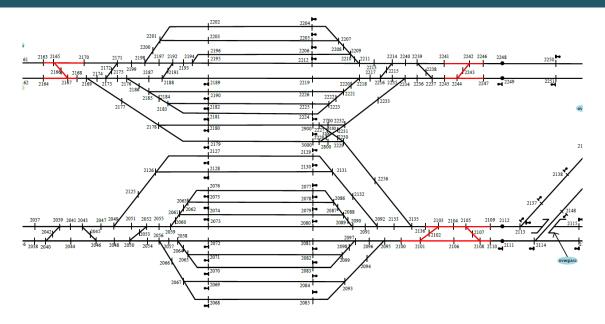


Train link running time assumption

When the running time of the train on each link is fractional, it should be rounded up to the integral multiple of the unit time interval. It is noted that one second is used as the unit time interval in this research.



One kind of speed restriction for all of the cells with track maintenance tasks in the same block section



For the block section containing several cells with track maintenance tasks that a train passes through, it is assumed that if the train is the first or second train traveling through one of the cells after track maintenance on that cell has finished, then the train will be the first or second train traveling through all of the cells after track maintenance tasks on those cells has finished.



Objective Function

The model is aimed to minimize total running time of all trains:

- 1 total running time of the trains when they release the arrival block sections in destination station.
- 2 scheduled dwell time of the trains at destination station.
- **3** minimal running time of the trains on the last link of arrival block section which connects the destination node.



Train Moving Constraints

> The relationship between usage of block sections and entry and exit time of trains on those block sections

$$y_{r,b}^{entry} \le Mx_{r,b}, \ \forall r \in R, \forall b \in B_r \tag{1}$$

$$y_{r,b}^{exit} \le Mx_{r,b}, \ \forall r \in R, \forall b \in B_r \tag{2}$$

➤ Minimal running time constraints

$$y_{r,b}^{exit} \ge y_{r,b}^{entry} + x_{r,b} \sum_{l \in L_b} t_{r,l}, \ \forall r \in R, \forall b \in B_r \setminus \left(B^{arrival} \cup B_n^+\right): \ n = n_r^o, n \notin N^{boundary}$$
(3)

$$\sum_{b \in B_n^+ \cap B_r} y_{r,b}^{exit} \ge \sum_{b \in B_n^+ \cap B_r} y_{r,b}^{entry} + \sum_{b \in B_n^+ \cap B_r} x_{r,b} \sum_{l \in L_b} t_{r,l} + stop_{r,s}, \ \forall r \in R: \ n = n_r^o, n \notin N^{boundary}$$

$$\tag{4}$$

$$y_{r,b}^{exit} \ge y_{r,b}^{entry} + x_{r,b} \sum_{l \in L_b \setminus c_b} t_{r,l}, \ \forall r \in R, \forall b \in B_r \cap B^{arrival}$$

$$\tag{5}$$



Train Moving Constraints

> Departure time window constraints

$$\sum_{b \in B_n^+ \cap B_r} y_{r,b}^{entry} \le t_r^e, \ \forall r \in R, n = n_r^o, \forall b \in B_n^+$$
 (6)

$$\sum_{b \in B_n^+ \cap B_r} y_{r,b}^{entry} \ge t_r^s, \ \forall r \in R, n = n_r^o, \forall b \in B_n^+$$
 (7)

> Entry and exit time cohesive relationship between two adjacent block sections

$$\sum_{b' \in B_n^- \cap B_r} y_{r,b'}^{exit} = \sum_{b \in B_n^+ \cap B_r} y_{r,b}^{entry}, \ \forall r \in R, \forall n \in N_r \setminus ((N^{siding} \cup N^{main}) \setminus N_r^{special}) : n \neq n_r^o, n \neq n_r^o$$

$$(8)$$

$$\begin{split} & \sum_{n \in N_{s} \cap \text{ ((N^{siding} \cup N^{main}) } \setminus N_{r}^{special})} \sum_{b' \in B_{n}^{-} \cap B_{r}} y_{r,b'}^{exit} + stop_{r,s} + \\ & \sum_{n \in N_{s} \cap \text{ ((N^{siding} \cup N^{main}) } \setminus N_{r}^{special})} \sum_{b' \in B_{n}^{-} \cap B_{r}} x_{r,b'} t_{r,c_{b'}} = \sum_{n \in N_{s} \cap \text{ ((N^{siding} \cup N^{main}) } \setminus N_{r}^{special})} \sum_{b \in B_{n}^{+} \cap B_{r}} y_{r,b}^{entry}, \\ & \forall r \in R, \forall s \in S_{r} \cup s_{r}^{special} \end{split}$$

As for the arrival block section and the departure block section, release time of the arrival block section and scheduled dwell time of trains should be considered.



Train Moving Constraints

➤ Minimal dwelling time constraints

$$stop_{r,s} \ge dwell_{r,s}, \ \forall r \in R, \forall s \in S^r \cup s_r^o \cup s_r^d$$
 (10)

> Trains stop constraints

$$stop_{r,s} \le (1 - x_{r,b})M, \ \forall r \in R, \forall n \in N^{main}, \forall b \in B_r \cap B_n^-: s = s^n, s \in (S^r \cup s_r^o \cup s_r^d)$$
 (11)



Block Section Selection Constraints

> Only one block section can be selected at the origin and destination node

$$\sum_{b \in B_n^+ \cap B_r} x_{r,b} = 1, \ \forall r \in R, n = n_r^o$$
 (12)

$$\sum_{b \in B_n^- \cap B_r} x_{r,b} = 1, \ \forall r \in R, n = n_r^d$$

$$\tag{13}$$

> Train flow conservation constraints at the intermediate nodes

$$\sum_{b \in B_n^+ \cap B_r} x_{r,b} = \sum_{b \in B_n^- \cap B_r} x_{r,b}, \ \forall r \in R, \forall n \in N : n \neq n_r^o, n \neq n_r^d$$

$$\tag{14}$$

➤ A train can only choose one block section traveling into the station

$$\sum_{n \in N_s \cap (N^{siding} \cup N^{main})} \sum_{b' \in B_n^- \cap B_r} x_{r,b'} = 1, \ \forall r \in R, \forall s \in S_r$$
 (15)



Block Section Occupancy Constraints

Two trains choose two conflicting block sections: two arrival block sections with different destination nodes, two different departure block sections with different origin nodes, one arrival block section and one departure block section

$$M \big(1 - x_{r',b'} \big) \, + \, M \big(1 - x_{r,b} \big) \, + \, y_{r',b'}^{entry} - y_{r,b}^{exit} \geq - M \big(1 - \mu_{r,b,r',b'} \big), \ \forall r,r' \in R, \forall n_1,n_2 \in \big(N^{siding} \cup N^{siding}$$

$$M\big(1-x_{r',b'}\big) + M\big(1-x_{r,b}\big) + y_{r,b}^{entry} - y_{r',b'}^{exit} \geq -M\mu_{r,b,r',b'}, \ \forall r,r' \in R, \forall n_1,n_2 \in \big(N^{siding} \cup N^{siding} \cup N^$$

> Two trains choose one same block section

$$M(1 - x_{r',b}) + M(1 - x_{r,b}) + y_{r',b}^{entry} - y_{r,b}^{exit} \ge -M(1 - \mu_{r,b,r',b}), \ \forall r,r' \in R, \forall b \in B_r \cap B_{r'} : r \ne r'$$
(18)

$$M(1 - x_{r',b}) + M(1 - x_{r,b}) + y_{r,b}^{entry} - y_{r',b}^{exit} \ge -M\mu_{r,b,r',b}, \ \forall r,r' \in R, \forall b \in B_r \cap B_{r'} : r \ne r' \ (19)$$



Block Section Occupancy Constraints

Sequences of trains on passing block sections can not be changed

$$\mu_{r,b,r',b} = \mu_{r,b',r',b'}, \ \forall r,r' \in R, \forall i \in I, \forall b \ , b' \in B_r \cap B_{r'} \cap B_i \setminus B^{share} : r \neq r', b' = b + 1$$
 (20)

> Sequences of trains on conflicting arrival block sections which have same destination node

$$M(1 - x_{r',b'}) + M(1 - x_{r,b}) + y_{r',b'}^{entry} - y_{r,b}^{exit} \ge stop_{r,s} + t_{r,c_b} - M(1 - \mu_{r,b,r',b'}), \ \forall r, r' \in R, n \in N^{siding} \cup N^{main}, \forall b \in B_r \cap B_n^-, b' \in B_{r'} \cap B_n^-: r \ne r', s = s^n, s \in (S^r \cup s_r^o \cup s_r^d)$$
(21)

$$M(1 - x_{r',b'}) + M(1 - x_{r,b}) + y_{r,b}^{entry} - y_{r',b'}^{exit} \ge stop_{r',s} + t_{r',c_{b'}} - M\mu_{r,b,r',b'}, \ \forall r,r' \in R, n \in N^{siding} \cup N^{main}, \forall b \in B_r \cap B_n^-, b' \in B_{r'} \cap B_n^-: r \ne r', s = s^n, s \in (S^{r'} \cup s_{r'}^o \cup s_{r'}^d)$$
(22)



Block Section Occupancy Constraints

> Origin node of the train is at the departure signal and other trains can not use the occupied main track or siding track until the train leaves the origin node

$$M(1-x_{r',b'})+y_{r',b'}^{entry} \geq \sum_{b \in B_n^+ \cap B_r} y_{r,b}^{entry} + stop_{r,s}, \ \forall r,r' \in R, s=s_r^o, \forall b' \in B_{r'} \cap B_n^-: r \neq r', n=n_r^o, n \notin N_r^{special}, s \neq s_r^{special}$$

$$(23)$$

> Origin node of the train is at the node behind the departure signal and other trains can not use the occupied main track or siding track until the train arrives at the departure signal

$$M\left(1-x_{r',b'}\right)+y_{r',b'}^{entry} \geq \sum_{b \in B_n^+ \cap B_r} y_{r,b}^{exit}, \ \forall r,r' \in R, s=s_r^o, \forall b' \in B_{r'} \cap B_n^-: r \neq r', n=n_r^o, n \in N_r^{special}, s \neq s_r^{special}$$

$$(24)$$



Block Section Occupancy Constraints

> Origin node of the train is at the boundary point and other trains using different arrival block sections should wait until the occupied arrival block section is released

$$M(1 - x_{r',b'}) + M(1 - x_{r,b}) + y_{r',b'}^{entry} \ge y_{r,b}^{exit}, \ \forall r, r' \in R, \ s^n = s_r^{special}, \forall b \in B_r \cap B_n^+, \forall b' \in B_{r'} \cap B_n^+: r \ne r', n = n_r^o, b \ne b'$$
(25)

> Origin node of the train is at the boundary point and other trains using same arrival block sections should wait until the train leaves the station

$$M(1 - x_{r',b}) + M(1 - x_{r,b}) + y_{r',b}^{entry} \ge y_{r,b}^{exit} + stop_{r,s}, \ \forall r,r' \in R, \ s^n = s_r^{special}, \forall b \in B_r \cap B_{r'} \cap B_n^+: r \ne r', n = n_r^o$$
(26)



Maintenance of Track Constraints

> MOT starting time window constraints

$$start_{c} \geq mot_{m}^{s}, \ \forall m \in MOT, \forall c \in C_{m}$$

$$start_{c} \leq mot_{m}^{e}, \ \forall m \in MOT, \forall c \in C_{m}$$

$$end_{c} - start_{c} \geq d_{m}, \ \forall m \in MOT, \forall c \in C_{m}$$

$$(28)$$

▶ Maintenance task entrance constraints

$$y_{r,b}^{exit} \le start_c + M(1 - x_{r,b}) + M\alpha_{r,b,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in B_r \cap B_c$$
 (30)

$$M(1-x_{r,b}) + y_{r,b}^{entry} \ge end_c - M(1-\alpha_{r,b,c}), \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in B_r \cap B_c$$
 (31)

➤ Maintenance task adjacency constraints

$$end_{c_2} \ge start_{c_1}, \ \forall m \in MOT, \forall c_1 \in C_m, \forall c_2 \in MOT_{c_1}^{adjacent}$$
 (32)

$$end_{c_1} \ge start_{c_2}, \ \forall m \in MOT, \forall c_1 \in C_m, \forall c_2 \in MOT_{c_1}^{adjacent}$$
 (33)



Speed Restriction Constraints

> Only speed of trains traveling through the MOT needs to be restricted

$$z_{r,c} \le \sum_{b \in B_r \cap B_c} x_{r,b}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$
(34)

> The relationship between train entry time and MOT end time

$$\sum_{b \in B_r \cap B_c} y_{r,b}^{entry} + M(1 - z_{r,c}) \ge end_c, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$
(35)

$$\sum_{b \in B_r \cap B_c} y_{r,b}^{entry} \le end_c - 1 + Mz_{r,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$
 (36)

➤ The first and second train traveling through the MOT must travel through the MOT after the MOT is finished

$$z_{r,c}^{1} \le z_{r,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_{m}$$

$$(37)$$

$$z_{r,c}^2 \le z_{r,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$

$$\tag{38}$$

> Speed restriction condition for the first train traveling through the MOT after the MOT is finished

$$M(1 - z_{r_1,c}^1) + M(1 - z_{r_1,c}) + M(1 - z_{r_2,c}) + \sum_{b \in B_{r_2} \cap B_c} y_{r_2,b}^{entry} \ge \sum_{b \in B_{r_1} \cap B_c} y_{r_1,b}^{entry}, \ \forall r_1, r_2 \in R, \forall m \in MOT, \forall c \in C_m: r_1 \ne r_2$$

$$(39)$$



Speed Restriction Constraints

➤ If there are trains traveling through the MOT after the MOT is finished, then the first train whose speed needs to be restricted exits

$$\sum_{r \in R} z_{r,c} \le M \sum_{r \in R} z_{r,c}^1, \ \forall m \in MOT, \forall c \in C_m$$

$$\tag{40}$$

> Entry time of the second train into the MOT is greater than or equal to the first train

$$M(1 - z_{r_1,c}^2) + Mz_{r_2,c}^1 + M(1 - z_{r_1,c}) + M(1 - z_{r_2,c}) + \sum_{b \in B_{r_2} \cap B_c} y_{r_2,b}^{entry} \ge \sum_{b \in B_{r_1} \cap B_c} y_{r_1,b}^{entry}, \ \forall r_1, r_2 \in R, \forall m \in MOT, \forall c \in C_m: r_1 \ne r_2$$

$$(41)$$

➤ A train can only be either the first train or the second train traveling through the MOT after the MOT is finished

$$z_{r,c}^1 + z_{r,c}^2 \le 1, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$

$$\tag{42}$$

➤ If there are at least two trains traveling through the MOT after the MOT is finished, then the second train whose speed needs to be restricted exits

$$\sum_{r \in R} z_{r,c} - 1 \le M \sum_{r \in R} z_{r,c}^2, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$$

$$\tag{43}$$



Speed Restriction Constraints

> Speed restriction constraints for the first train

$$\sum_{c \in C_b \cap C_m} M(1 - z_{r,c}^1) + y_{r,b}^{exit} \ge y_{r,b}^{entry} + x_{r,b} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c} {l_r^{\nu}} / \min_{\{l_r^{\nu}, v_c^1\}} t_{r,l} + \sum_{l \in L_b \cup L_c (c \in C_m)} t_{r,l} \right),$$

$$\forall r \in R, \forall m \in MOT, \forall b \in B_r \backslash B_{arrival} : C_b \cap C_m \ne \emptyset$$

$$\sum_{c \in C_b \cap C_m} M(1 - z_{r,c}^1) + y_{r,b}^{exit} \ge y_{r,b}^{entry} + x_{r,b} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \backslash C_b} {l_r^{\nu}} / \min_{\{l_r^{\nu}, v_c^1\}} t_{r,l} + \right)$$

$$(44)$$

> Speed restriction constraints for the second train

$$\sum_{c \in C_b \cap C_m} M \left(1 - z_{r,c}^2 \right) + y_{r,b}^{exit} \ge y_{r,b}^{entry} + x_{r,b} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c} \frac{l_r^v}{r} / \min_{\{l_r^v, v_c^2\}} t_{r,l} + \frac{l_r^v}{r} \right)$$

$$\textstyle \sum_{c \in C_b \cap C_m} M \left(1 - z_{r,c}^2\right) + y_{r,b}^{exit} \geq y_{r,b}^{entry} + x_{r,b} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \big/_{\min\{l_r^v, v_c^2\}} t_{r,l} + \frac{1}{2} \left(\sum_{c \in C_b \cap C_m} \sum_{l \in L_b \cap L_c \setminus c_b} l_r^v \right) \right)$$

> One kind of speed restriction for all of the cells with track maintenance tasks in the same block section

$$z_{r,c}^1 = z_{r,c'}^1, \ \forall r \in R, \forall m \in MOT, \forall b \in B_r, c \in C_b \cap C_m : c' = c+1, card(\ C_b \cap C_m) \geq 2 \tag{48}$$

$$z_{r,c}^2 = z_{r,c'}^2, \ \forall r \in R, \forall m \in MOT, \forall b \in B_r, c \in C_b \cap C_m : c' = c + 1, card(C_b \cap C_m) \ge 2$$

$$\tag{49}$$



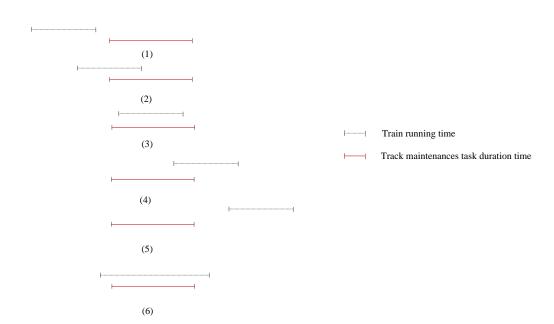
Speed Reduction Constraints

$$y_{r,b}^{entry} \ge end_c - M(1 - \alpha_{r,b,c}), \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c$$
 (50)

$$y_{r,b}^{entry} \le end_c + M\alpha_{r,b,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c$$
 (51)

$$start_c \geq y_{r,b}^{entry} + \sum_{l \in L_b} t_{r,l} - M(1 - \beta_{r,b,c}), \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c \ (52)$$

$$start_c \le y_{r,b}^{entry} + \sum_{l \in L_b} t_{r,l} + M\beta_{r,b,c}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c$$
 (53)





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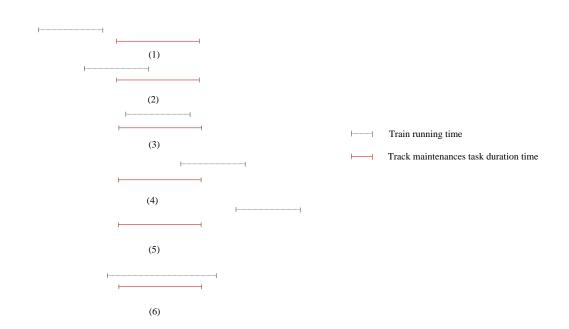
Speed Reduction Constraints

> Speed of trains in situation (2), (3) and (4) needs to be reduced

$$\alpha_{r,b,c} + \beta_{r,b,c} \le 1, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c \tag{54}$$

$$M\alpha_{r,b,c} + M\beta_{r,b,c} + y_{r,b}^{exit} \ge y_{r,b}^{entry} + \sum_{l \in L_b} {l_r^v} / \min\{l_r^v, v_c^{limit}\} t_{r,l}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_i): i = i_c$$

$$(55)$$





Values of Variables

$y_{r,b}^{entry}, y_{r,b}^{exit} \in N, \ \forall r \in R, \forall b \in B_r$	(56)
$x_{r,b} \in \{0,1\}, \ \forall r \in R, \forall b \in B_r$	(57)
$\mu_{r,b,r',b'} \in \{0,1\}, \ \forall r \in R, \forall b \in B_r, \forall r' \in R, \forall b \in B_{r'} \colon r \neq r'$	(58)
$stop_{r,s} \in N, \ \forall r \in R, \forall s \in S^r \cup s_r^o \cup s_r^d$	(59)
$start_c, end_c \in N, \ \forall m \in MOT, \forall c \in C_m$	(60)
$\alpha_{r,b,c},\beta_{r,b,c} \in \{0,1\}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m, \forall b \in (B_r \cap B_c)$	(61)
$z_{r,c}, z_{r,c}^1, z_{r,c}^2 \in \{0,1\}, \ \forall r \in R, \forall m \in MOT, \forall c \in C_m$	(62)

An Integrated Optimization Method for Train Timetabling and Maintenance Scheduling Problem



3 Solution Approaches



3.1 Simplification of Objective Function

minimize
$$Z = \sum_{r \in R} \left(\sum_{b \in B_n^- \cap B_r, n = n_r^d} y_{r,b}^{exit} - \sum_{b \in B_n^+ \cap B_r, n = n_r^o} y_{r,b}^{entry} \right)$$
 (63)

The objective function of our research consists of three parts, while two of the three parts corresponding to scheduled dwell time of the trains at destination station and minimal running time of the trains on the last link of arrival block section which connects the destination node turn out to be constants, which will not affect the optimization results of the model. Hence, the object function is simplified as expression (63).



3.2 Analysis on the Value of Big M

$$y_{r,b}^{entry} \le Mx_{r,b}, \ \forall r \in R, \forall b \in B_r \tag{1}$$

$$y_{r,b}^{exit} \le Mx_{r,b}, \ \forall r \in R, \forall b \in B_r \tag{2}$$

$$stop_{r,s} \le (1 - x_{r,b})M, \ \forall r \in R, \forall n \in N^{main}, \forall b \in B_r \cap B_n^-: s = s^n, s \in (S^r \cup s_r^o \cup s_r^d)$$
 (11)

The value of big M is critical for several constraints. Generally, the smaller the value of big M is, the higher is the solving efficiency (Yan and Yang, 2011).

However, in constraint (1), (2), and (11), the value of big M will also affect the value of related variables. For instance, in constraint (1) and (2), the value of big M will affect the possible entry and exit time of block sections. So, under the condition of not changing the solving quality, we hope the value of big M is as small as possible to obtain a better efficiency.



3.3 Sequence of Trains on the Passing Block Sections

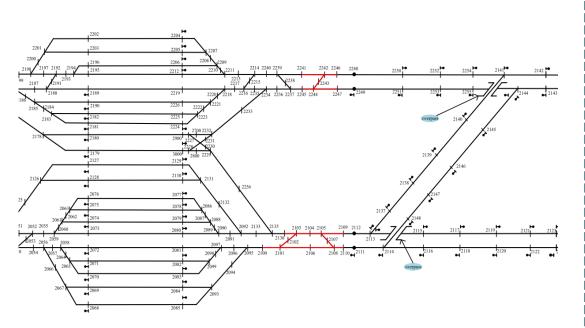


Figure 4 Analysis on passing block sections used by trains nearby station M

Sequences of trains on the same passing block sections will not change. Therefore, in Expression (20), we set the value of decision variable $\mu_{r,b,r',b} = \mu_{r,b+1,r',b+1}$.

There are some branches around station M where passing block sections diverge or merge, and constraint (20) should not be applied to the passing block sections which are the first passing block section before the diverge or after the merge.



3.4 Analysis on the Value of $z_{r,c}$

If earliest entry time for train r on block section b is even larger than the sum of end_c , minimum duration time of track maintenance task on cell c and extra 5h, and cell c is contained in block section b, then we can set the value of $z_{r,c}$ to 1 in advance to improve the solving efficiency without affecting the solution space.

4 Case Study



Table 7 Solution results

	CPU Times (s)	Objective Value (s)	Gap (%)
Case 1	51.08	154361	0
Case 2	457.03	154367	0
Case 3	3041.91	154439	0.0094

- ➤ In Case 1, the optimum solution is acquired after 51.08s, and the gap is 0, which proved that our solution is optimal. This result indicates that the maintenance task will not affect the train movement. All trains have left station M before the maintenance tasks started.
- ➤ In Case 2, the number of maintenance tasks increases to 2, which make the problem more difficult to solve. The CPU time increases to 457.03s. There will be only one train affected by the speed restriction constraint which is the first train running on the cell after the track maintenance task has finished.
- ➤ In Case 3, the number of maintenance tasks increases to 4 and one of the maintenance tasks is located on the track between two stations, and this leads to the CPU time increases dramatically, which is up to 3041s with gap 0.0094%.

4 Case Study



Table 8 Solution results with revised maintenance task adjacency constraints

	CPU Times (s)	Objective Value (s)	Gap (%)
Case 1	30.74	154361	0
Case 2	188.06	154367	0
Case 3	3609.59	155255	0.577

- ➤ All cells included in one maintenance task have same start and end time, and only time window of adjacent maintenance tasks should be overlapping or contiguous.
- ➤ As for Case 1 and Case 2, CPU times decrease with the optimal objective values unchanged, which means the difficulty in solving those two cases has been reduced.
- As for Case 3, the total available CPU times are set to 3600 s, and the objective value with gap 0.577% is 816 s higher than the original model, which means the difficulty in solving Case 3 has been increased. Possible reason is that much harder block sections usage constraints are set in Case 3.

5 Conclusion



The mixed integer linear programming model aims to schedule each train on the link and maintenance task on cell with train moving constraints, block section selection constraints, block section occupancy constraints and maintenance task constraints.

Gurobi 6.5.2 can solve the model in case 1, case 2 and case 3 in a reasonable time, which is 51.08s for Case 1, 457s for Case 2, and 3041s for Case 3. But when the number of trains increases, a decomposition algorithm is needed to solve the problem.



Thanks for Listening!